Tracking Error of 100-m Antenna due to Wind Gust

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A procedure is shown to derive root-mean-square tracking error for an antenna system in response to wind-gust loading. Example calculations are illustrated for the design of a proposed 100-m antenna. The effect of wind-gust correlation is also considered.

I. Introduction

Wind loading is an important factor in the design of many above-ground structures. For large antenna structures in particular it is necessary to make some allowance for possible wind loading. Wind motion may be described as consisting of two parts: a mean motion and a superimposed turbulent fluctuation. Some studies have provided detailed measurements of the time history of wind fluctuations, and techniques of random process theory have been employed to gain insight into the physical processes controlling the turbulent fluctuations. This article defines and calculates the tracking error of a proposed 100-m-diameter antenna due to wind-gust torque as a function of equivalent spring constant.

II. Antenna Tracking Error

Wind-gust torques are analyzed as a random process with zero mean. For a system with torque input T_Z and angular displacement output θ_{rms}, (see Fig. 1), we define

T_Z = mean torque input due to wind, Nm

$S_T(f)$ = input spectral density for dynamic wind load

$Y(jf)$ = system admittance to wind torque

$S_0(f)$ = output spectral density

θ_{rms} = tracking error due to wind torque

j = the imaginary unit

The root-mean square (rms) value of the output is obtained from

$$\theta_{rms} = \left\{ \int_0^\infty S_0(f)\,df \right\}^{1/2} \tag{1}$$

where $S_0(f)$ can be computed by

$$S_0(f) = |Y(jf)|^2 S_T(f) \tag{2}$$

in which

$$Y(jf) = \frac{jf}{K_{eq} f_c \left[\left(j\frac{f}{f_n} \right)^2 + j2\zeta \left(\frac{f}{f_n} \right) + 1 \right] \left[j\left(\frac{f}{f_c} \right) + 1 \right]} \tag{3}$$

K_{eq} = series combination of structural stiffness and servo drive stiffness

f_c = servo position loop bandwidth, Hz

f_n = natural frequency of the structure, Hz

ζ = damping ratio, relative to critical damping

f = excitation frequency, Hz

To define the spectral density of the dynamic load, the spectrum of the velocity fluctuations is first required. The velocity spectral density used in this report was obtained by Davenport (Ref. 1), giving the spectrum in the form

$$S_V(f) = \frac{4K\,U_1^2\,X^2}{f(1+X^2)^{4/3}} \tag{4}$$

in which

$S_V(f)$ = velocity spectral density (one-sided)

U_1 = steady speed of wind at reference height of 10 meters, m/s

K = surface drag coefficient for the local terrain

$X = 1200\,f/U_1$

To relate the load spectrum to the velocity spectrum, the relation given by Vellozzi and Cohen (Ref. 2) is used, giving the gust-load spectrum in the form:

$$S_T(f) = \frac{4T_z^2}{V_z^2}\,S_V(f) \tag{5}$$

in which T_Z and V_Z are the steady load and mean wind speed at height Z (meters).

Since the wind gusts are not likely to act simultaneously over the full height of structure, a correlation function (Ref. 2), $C^2(f)$, will be introduced in Eq. (5), which is therefore rewritten as

$$S_T(f) = \frac{4T_Z^2}{V_Z^2}\,S_V(f)C^2(f) \tag{6}$$

in which

$$\left.\begin{aligned} C^2(f) = &\left\{\frac{1}{\xi} - \frac{1}{2\xi^2}(1-e^{-2\xi})\right\} \\ &\times\left\{\frac{1}{\gamma} - \frac{1}{2\gamma^2}(1-e^{-2\gamma})\right\} \\ &\times\left\{\frac{1}{\mu} - \frac{1}{2\mu^2}(1-e^{-2\mu})\right\} \end{aligned}\right\} \tag{7}$$

where

$$\xi = \frac{3.85\,f\Delta X}{U},\ \gamma = \frac{11.5\,f\Delta Y}{U},\ \mu = \frac{3.85\,f\Delta Z}{U} \tag{8}$$

and

$$U = \frac{U_1}{(1+\alpha)}\left(\frac{h}{10}\right)^{\alpha} \tag{9}$$

in which ΔX, ΔY, and ΔZ = the alongwind, crosswind, and vertical distances over which the correlation is being taken, h is the height of structure, and α is the exponent in the assumed power low-speed profile of the wind vs height of the structure relationship given by

$$U_1 = V_Z\left(\frac{10}{Z}\right)^{\alpha} \tag{10}$$

A. Calculations

To develop Eq. (4) specifically for the case at hand, the following parameters are used:

$$\left.\begin{aligned}\alpha &= 0.1405 \\ K &= 0.005\end{aligned}\right\}\ \text{for Goldstone site (Ref. 3)}$$

$$Z = 48\text{ m}$$

then from Eq. (10),

$$U_1 = V_Z\left(\frac{10}{48}\right)^{0.1405} \quad \text{or} \quad U_1 = 0.80\,V_Z$$

with these relationships, the specific representation of Eq. (4) becomes

$$S_V(f) = \frac{28800f}{(1+X^2)^{4/3}}$$

For example, with $V_Z = 4.47$ m/s,

$$U_1 = 0.80\,(4.47) = 3.58 \text{ m/s}$$

Thus

$$X = 1200 \frac{f}{3.58} = 335.57\,f$$

and the gust velocity spectrum is given by

$$S_V(f) = \frac{28800\,f}{(1 + 112608\,f^2)^{4/3}}$$

Figure 2 shows wind velocity spectral density curves for a family of values of mean speeds, V_Z. Table 1 gives the spectral density equations derived to plot the curves. The tabulated values of f_{max} and $S_V(f_{max})$ indicate the frequency at which the spectra peak and the associated magnitude, respectively.

B. Derivation of the Wind Torque

Loading data derived from existing antenna wind tunnel tests was applied to an analytical model of the 100-m antenna. The antenna orientation shown in Fig. 3 results in the maximum wind torque on the antenna dish, at a speed of 31.29 m/s (70 mph) the result was

$$T_Z = 5.347 \times 10^7 \text{ Nm at } 31.29 \text{ m/s}$$

$$(3.951 \times 10^7 \text{ lb-ft at } 70 \text{ mph})$$

C. Calculation of Wind Torque for Different Speeds

To calculate wind torque for different speeds of wind, the following formula expresses the relationship that torque is proportional to the square of wind speed, e.g.,

$$T = \left(\frac{V}{V_{ref}}\right)^2 T_{ref} \tag{11}$$

where T_{ref} and V_{ref} are reference torque and speed, respectively.

Table 2 shows the results of Eq. (11) applied to several mean speeds using the reference speed and torque from the wind tunnel analysis.

D. Calculation of K_{eq}

The equivalent spring constant is computed by means of the well-known relationship for springs in series

$$\frac{1}{K_{eq}} = \frac{1}{K_{structure}} + \frac{1}{K_{servo}} \tag{12}$$

From structural and servo system analysis, the following spring constants were derived:

$$K_{structure} = 2.82 \times 10^{10} \frac{\text{Nm}}{\text{rad}}$$

$$K_{servo} = 6.78 \times 10^{10} \frac{\text{Nm}}{\text{rad}}$$

substituting these values in Eq. (12) provides

$$K_{eq} = 1.99 \times 10^{10} \frac{\text{Nm}}{\text{rad}}$$

E. Calculation of the Tracking Error

The digital computer was used to integrate Eq. (1) numerically and calculate θ_{rms}. Example results are listed as follows:

Case 1: $f_c = 0.20$ Hz, $f_n = 1.00$ Hz, $\zeta = 0.01$

$C^2(f) = 1$ (100% correlation)

Table 3 shows the constants that can be used to compute the tracking error for Case 1. The tracking error is obtained by dividing the constant by K_{eq}. The results of Case 1 are plotted in Fig. 4.

Case 2: $f_c = 0.20$ Hz, $f_n = 1.00$ Hz, $\zeta = 0.01$

$$C^2(f): \begin{bmatrix} h = 104 \text{ meters}, \Delta X = 50 \text{ meters} \\ \Delta Y = 50 \text{ meters}, \Delta Z = 104 \text{ meters} \end{bmatrix}$$

Notes h and ΔZ are the overall height of the antenna at stow position.

Table 4 shows the tracking error constants for Case 2. The results of Case 2 are plotted in Fig. 5.

III. Conclusion

The tracking error of the proposed 100-m-diameter antenna due to wind gust has been defined and calculated. In Case 1, in which wind gust has 100 percent correlation over the antenna structure, the tracking error is about 0.012 deg. at 13.14 m/s (30 mph) compared to 0.00043 deg. of Case 2, which is a more realistic result in terms of wind-gust effects on the antenna.

Of course, in considering the above results, one should bear in mind that a reasonable tolerance margin must be taken into account, and these results merely give some understanding to the design team about the antenna performance with respect to wind gust.

References

1. Davenport, A. G., "The Spectrum of Horizontal Gustiness Near the Ground in High Winds," *Quarterly Journal of the Royal Meteorological Society*, London, Vol. 87, Aug. 1961, pp. 194-211.

2. Velozzi, J., and Cohen, E. "Gust Response Factors", *Journal of the Structural Division, Proceedings of the American Society of Civil Engineers*, F ASCE, June 1968.

3. *Wind Power Prediction Models*, Technical Memorandum 33-802, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California, November 15, 1976.

Table 1. Family of wind velocity spectral densities

V_Z, m/s	$S_V(f)$, $(\mathrm{m/s})^2/\mathrm{Hz}$	f_{max}, Hz	$S_V(f_{max})$, $(\mathrm{m/s})^2/\mathrm{Hz}$
4.47	$\dfrac{28800f}{(1 + 112608f^2)^{4/3}}$	2.31×10^{-3}	35.52
8.94	$\dfrac{28800f}{(1 + 28152f^2)^{4/3}}$	4.62×10^{-3}	71.05
13.41	$\dfrac{28800f}{(1 + 12512f^2)^{4/3}}$	6.92×10^{-3}	106.57
17.88	$\dfrac{28800f}{(1 + 7038f^2)^{4/3}}$	9.23×10^{-3}	142.10
22.35	$\dfrac{28800f}{(1 + 4504f^2)^{4/3}}$	1.15×10^{-2}	177.63
26.82	$\dfrac{28800f}{(1 + 3128f^2)^{4/3}}$	1.38×10^{-2}	213.14
35.76	$\dfrac{28800f}{(1 + 1759f^2)^{4/3}}$	1.85×10^{-2}	284.23
44.70	$\dfrac{28800f}{(1 + 1126f^2)^{4/3}}$	2.31×10^{-2}	355.25
53.64	$\dfrac{28800f}{(1 + 782f^2)^{4/3}}$	2.77×10^{-2}	426.29

Table 2. Wind torque for different speeds

V_Z, m/s	T_Z, Nm
4.47	1.09×10^6
8.94	4.37×10^6
13.41	9.84×10^6
17.88	1.75×10^7
22.35	2.73×10^7

Table 3. Case 1 tracking error

V_Z, m/s	θ_{rms} ($\times K_{eq}$), deg
4.47	(1.81×10^7)
8.94	(9.14×10^7)
13.41	(2.36×10^8)
17.88	(4.61×10^8)
22.35	(7.75×10^8)

Table 4. Case 2 tracking error

V_Z, m/s	θ_{rms} ($\times K_{eq}$), deg
4.47	(3.24×10^5)
8.94	(2.56×10^6)
13.41	(8.51×10^6)
17.88	(1.99×10^7)
22.35	(3.83×10^7)

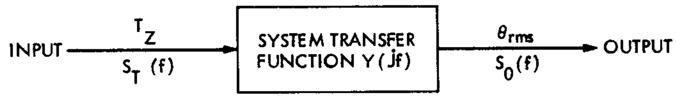

Fig. 1. System block diagram

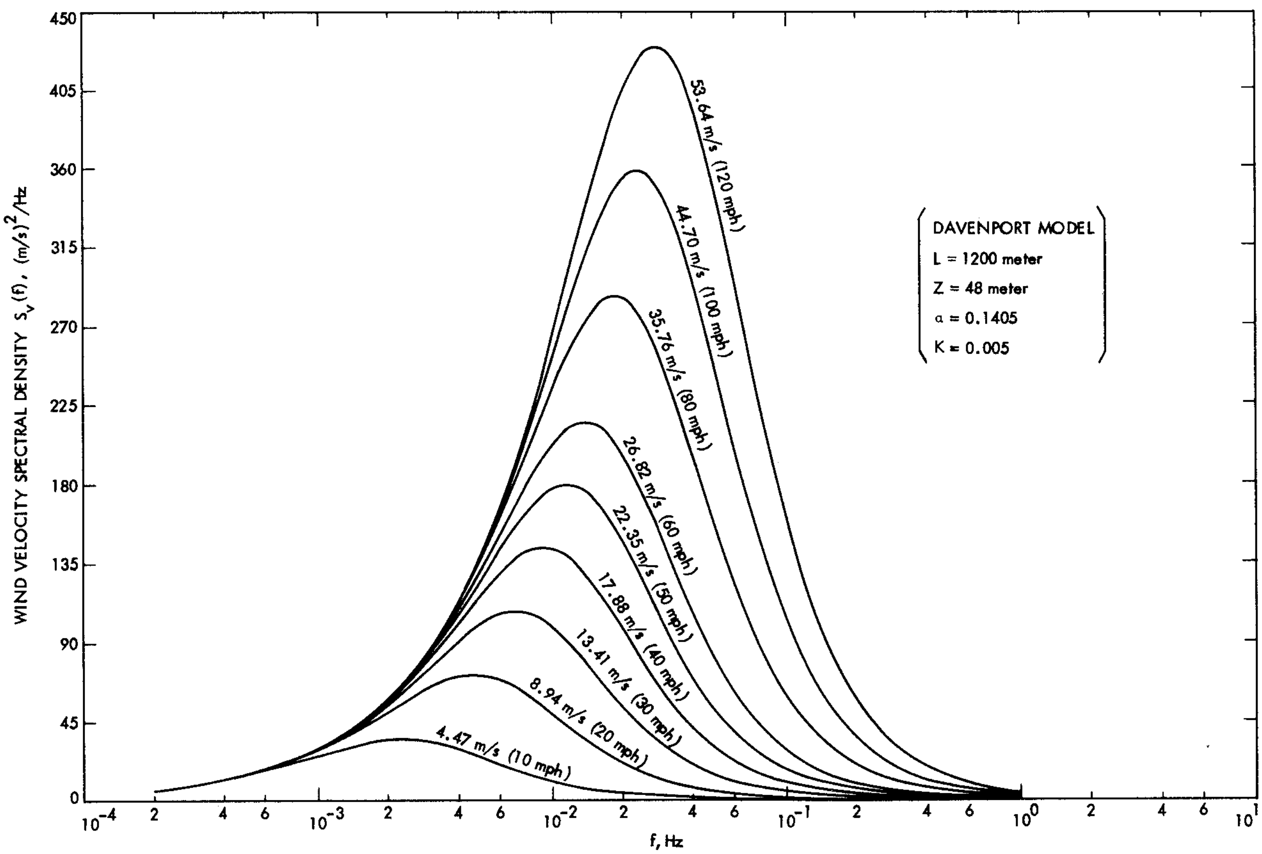

Fig. 2. Wind velocity spectral density

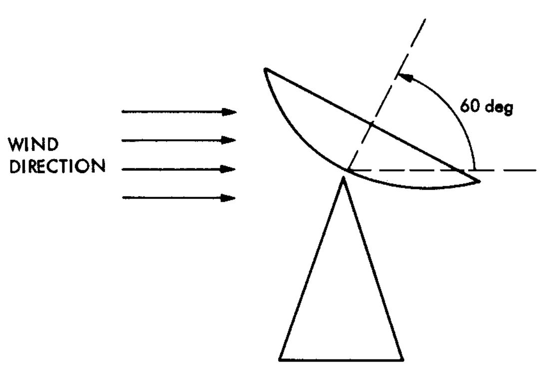

Fig. 3. Antenna orientation diagram

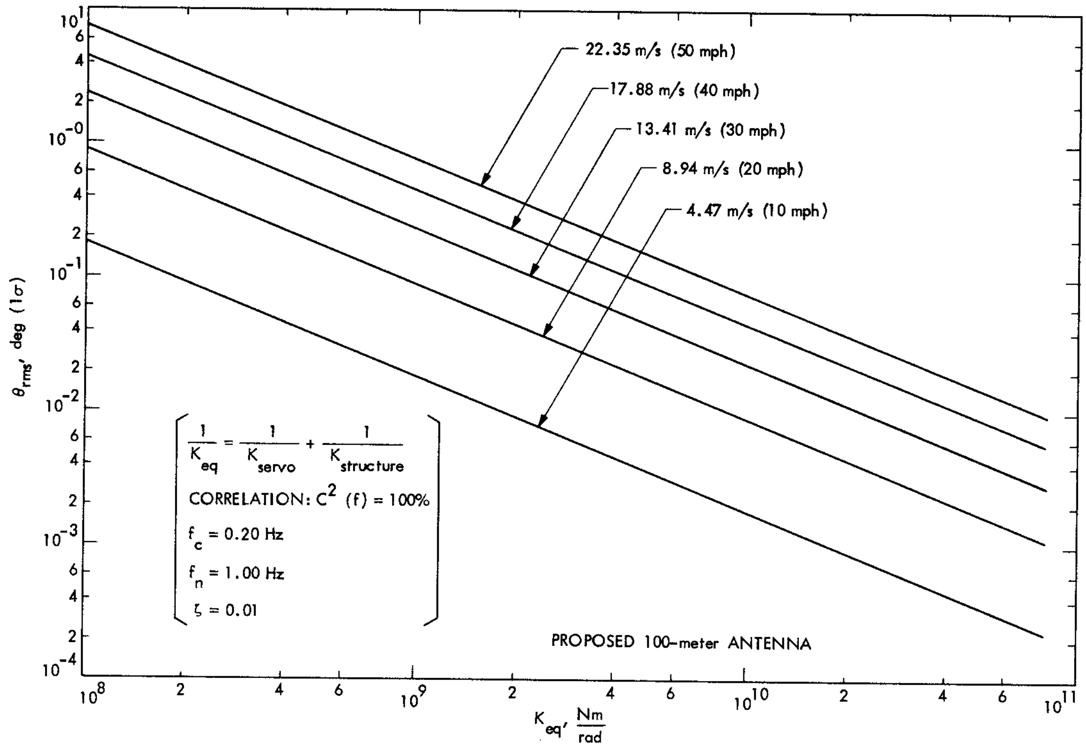

Fig. 4. Tracking error vs equivalent spring constant, Case 1, proposed 100-m antenna

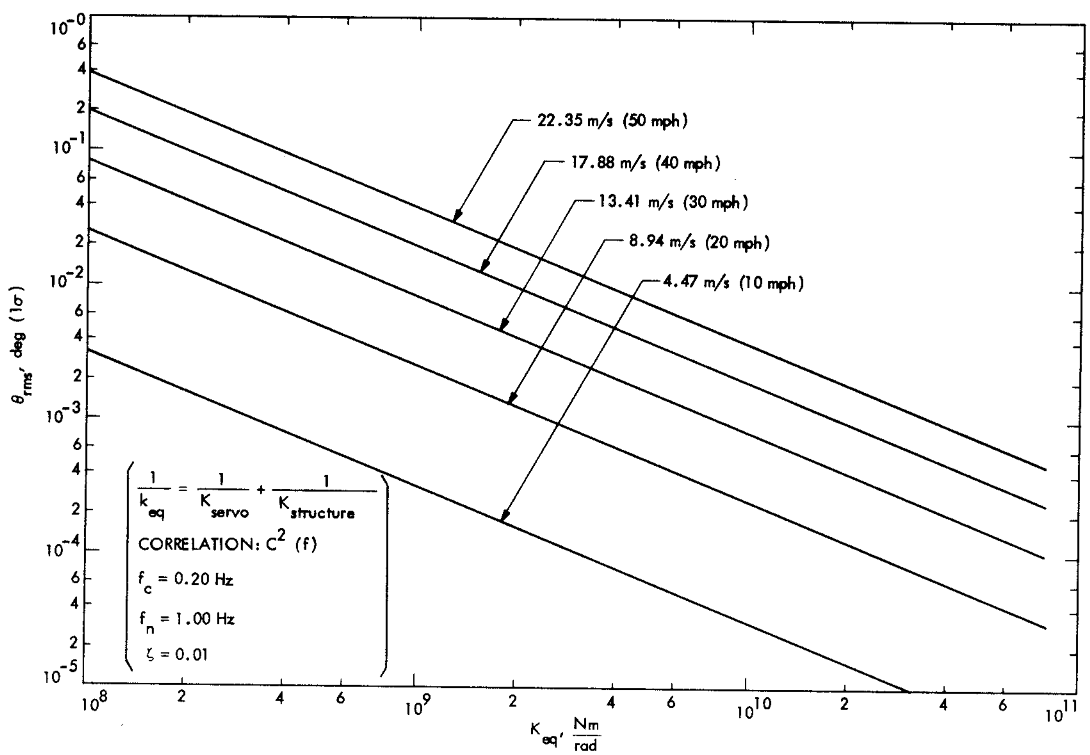

Fig. 5. Tracking error vs equivalent spring constant, Case 2, proposed 100-m antenna